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COMMENT

Critical dynamics for one-dimensional models

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Abstract. A plausible argument using the concepts of dynamic renormalisation group is used to derive the dynamical exponent z of the one-dimensional $q=2$, 3 and 4 Potts and four-state clock models. The exact result ($z=2$) for the Ising model with Glauber dynamics is recovered and in addition we find that the $q=3$, 4 Potts and four-state clock models belong to the same universality class. Moreover we find that $z=3.05\pm0.1$ for the ($q=2$) Potts model with Kawasaki dynamics in agreement with the $z=4-\eta$ prediction of Halperin, Hohenberg and Ma. A simple qualitative derivation is also given.

The purpose of this comment is to determine the dynamic exponent, z, of the one-dimensional q-state Potts model with Glauber (i.e. non-conserved order parameter) and Kawasaki (conserved order parameter) dynamics. It will be seen that the exact result of Glauber (1963) and Suzuki and Kubo (1968) for the Ising model with Glauber dynamics is recovered and that the $q=3$, 4 Potts models and the four-state clock model are all in the same dynamic universal class. However, with Kawasaki dynamics we find that $z=3$ in agreement with the prediction of $z=4-\eta$ made by Halperin *et al* (1974).

Consider in the spirit of Jan *et al* (1983) and Kalle (1984) the following arrangement. The one-dimensional spin system is initially set in a completely disordered state (equivalent to a high temperature state) and the system is quenched to the temperature $T_c=0$ (the critical temperature of the one-dimensional Ising chain with nearest-neighbour interactions) and allowed to evolve with Glauber dynamics. Qualitatively, it is clear that the domains of aligned spins will eventually grow and the size L of these domains will increase as

$$L\sim t^{1/z} \tag{1}$$

since as observed by Jan *et al* (1983) and Stauffer (1984) a renormalisation of length by a factor of b entails a renormalisation of time by a factor of b^z at T_c. Here t is measured in units of Monte Carlo steps per spin (MCS).

We need to consider, to evaluate z, the growth of domains due to the coalescence of domain walls. These perform a random walk and whenever they meet their number is irreversibly reduced by two, i.e. the system can be described by the reaction scheme

$$A+A\to\lambda A \tag{2}$$

where λ is some number between 0 and $2-\varepsilon$. However, considerable work has been done on such systems (see e.g. Toussaint and Wilczek (1983) and also Torney and

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McConnell (1983a, b)). Their conclusion is that the number of particles $N(t)$ decays as $t^{-1/2}$. Hence one has $L(t) \sim N(t)^{-1} \sim t^2$, bringing the result $z = 2$ for the q-state Potts model, which was known to be exact for $q = 2$. We have checked this result with extensive computer simulation and have confirmed the above value for $q = 3$ and 4. We also considered the four-state clock model where intermediate energy states may influence the relaxation of the system and hence the dynamic exponent, but apart from initial transients we observe the same universal behaviour.

For Kawasaki dynamics the situation is quite different. It is readily seen that, at $T = 0$, any state without domains of length one is stable, because no spin exchange is energetically favourable. Consider, however, a very small but non-zero temperature. Then one can use as a basic time unit the number of MCS per spin necessary to bring about one successful spin exchange, i.e. the creation of a domain of length one. This timescale is of the order of $L\,e^{J/T}$ and hence infinitely long compared with the diffusion time of a domain of length one, which is of order L^2. (This inequality begins to break down when $L \sim e^{J/T} \sim \xi$, which is, of course, to be expected.) We therefore consider the diffusion of a particle of one type surrounded by particles of another type as instantaneous.

The process considered is, therefore, as follows. At each time step a spin at a randomly selected domain wall is exchanged with its neighbour of the opposite phase, after which it performs an instantaneous random walk until it reaches either its original domain or the next one. It is easy to calculate the probability, which is $1/L$, for this spin to reach the next cluster, thereby changing the configuration. In order to observe the successful transfer of such a spin we need to allow an average of L events. The net average flux from one domain to the next is clearly zero and thus we require L^2 successful transfers before a domain of size L disappears as the result of a fluctuation. Hence the total time taken for the completion of these events is L^2 times the inverse probability, L. Thus $L \sim t^{1/3}$ and $z = 3 = 4 - \eta$ in accordance with the classic result of Halperin *et al* (1974). Computer simulations confirm this prediction where we observe $z = 3.05 \pm 0.1$.

We summarise our results: one-dimensional systems with short-range interactions have a critical temperature at $T = 0$ and this property determines the dynamical exponent z. Hence we find that systems with Glauber dynamics belong to the same universality class ($z = 2$) and the Ising model with Kawasaki dynamics has an exponent $z = 3$. These results may easily be extended to quasilinear systems, e.g. the unbranched Koch curve where $z = 2d_f$ (Glauber dynamics) and $z = 3d_f$ (Kawasaki dynamics) where d_f is the fractal dimensionality of the Koch curve.

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